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Leading edge effect during transient buoyancy induced flow adjacent to a vertical cylinder

K. VELUSAMY
Thermal Hydraulics Section, Indira Gandhi Centre for Atomic Research, Kalpakkam 603102, India

and

VIJAY K. GARG
NASA Lewis Research Center, Mail Stop 5-11, Cleveland, OH 44135, U.S.A.

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Abstract—The rate of propagation of the leading edge effect (LEE) during transient natural convection adjacent to a vertical solid cylinder is estimated from five different criteria. The cylinder has an appreciable thermal capacity and is subjected to a sudden heat generation. Numerical results are presented for a wide range of cylinder radii and heat flux values for two fluids, air and water. It is found that unlike the case of a flat plate, there is no unique criterion which would always estimate the fastest rate of propagation of LEE in water. However in air, the criterion due to Brown and Riley (*J. Fluid Mech.* **59**, 225–237 (1973)) always predicts the fastest rate of propagation. Also, the effect of cylinder radius on the rate at which the LEE propagates through different fluids is different. For identical conditions, the LEE propagates faster in air than in water, as expected. Present results obtained by dropping the curvature terms in the governing equations match very well with previous analytical results for a flat plate.

1. INTRODUCTION

TRANSIENT natural convection is of great importance in many industrial applications such as in nuclear and electronics industries. It arises out of a sudden change in surface conditions such as heat flux and temperature. In any system, it takes a specific time for natural convection to set in. During the initial period, heat is transferred by pure conduction, and a one-dimensional form of boundary layer equations hold. However, this one-dimensional process breaks down on the arrival of the LEE. This effect is marked by the appearance of the cross-stream velocity component, and travels at a finite speed downstream. Subsequent to the arrival of LEE, only transient boundary layer equations hold.

Siegel [1] was perhaps the first to point out the effect of LEE and time duration of the one-dimensional conduction process for a flat plate, while the transient analysis was initiated by Illingworth [2]. A detailed literature survey on transient free convection adjacent to a flat plate and a vertical cylinder was recently provided by Velusamy and Garg [3]. Goldstein and Briggs [4] presented solutions for the duration of one-dimensional process from the conduction analysis of plates and cylinders. Their analysis includes various boundary conditions such as sudden change in surface temperature and surface heat flux. However, for a flat plate, Mollendorf and Gebhart [5] and Mahajan and Gebhart [6] found that the actual rate of propagation of LEE is about 20% faster than that predicted by Goldstein and Briggs [4]. Yang [7] and Nanbu [8] analyzed the transient boundary layer equations and concluded that the departure from the one-dimensional process occurs at a critical time when an essential singularity appears in the governing equations. Brown and Riley [9] pointed out that this critical time resulted in a leading edge propagation criterion different from that proposed by Goldstein and Briggs [4]. For flat plates, Joshi [10] compared four different propagation criteria and found that the propagation rate based on the criterion of 'no overshoot in the mass flow rate during the one-dimensional process' is the fastest, and usage of other criteria implies an unrealistic overshoot in the mass flow rate for a Boussinesq fluid. This was found to be in close agreement with experimental data in water.

Besides the recent analysis of Velusamy and Garg [3] transient solutions for cylinders were carried out by Goldstein and Briggs [4], and by Dring and Gebhart [11]. The latter authors presented experimental results for the transient average temperature of Nichrome wires in silicone oils and in air. They also compared their experimental results with the pure conduction results, and with a simplified quasi-static theory that yields a simple exponential solution for the temperature response. The quasi-static theory failed, however, for silicone fluids. Even for air, the conduction solution was found to be better than that predicted by this theory.

NOMENCLATURE

- c thermal capacity of the cylinder per unit surface area
- Cp_m specific heat of the cylinder material
- g acceleration due to gravity
- Gr_x modified local Grashof number
- k thermal conductivity of the fluid
- LEE leading edge effect
- Pr Prandtl number of the fluid
- q''' volumetric energy generation rate
- q'' instantaneous energy generation rate within the cylinder per unit surface area
- Q thermal capacity parameter
- r radial coordinate measured from the centerline of the cylinder
- r_o radius of the cylinder
- R dimensionless radial coordinate
- R_o fourth root of modified Grashof number with r_o as length
- R_∞ edge of the boundary layer
- t temperature of the fluid within the boundary layer
- T dimensionless temperature
- u, v velocity components in x, r directions, respectively
- U, V dimensionless velocity components in X, R directions, respectively
- x axial coordinate measured upward
- X dimensionless axial coordinate
- X_{BR} penetration distance of the leading edge effect based on equation (9)
- X_{GB} penetration distance of the leading edge effect based on equation (8)
- X_J penetration distance of the leading edge effect based on equation (10)
- X_S penetration distance of the leading edge effect based on equation (12)
- X_T penetration distance of the leading edge effect based on equation (13)
- X_U penetration distance of the leading edge effect based on equation (11).

Greek symbols

- α thermal diffusivity of the fluid
- β coefficient of volumetric expansion of the fluid
- ν kinematic viscosity of the fluid
- ρ_m density of the cylinder material
- τ dimensionless time
- $\hat{\tau}$ time
- ψ dummy variable.

Subscripts

- s value at the cylinder surface
- ss value at steady state
- ∞ value in the free-stream.

It is well known that the boundary layer over a slender cylinder is thicker than that over a flat plate. Hence the results for a flat plate do not apply directly to slender cylinders. Moreover, for solid cylinders, Velusamy and Garg [3] found that the criterion proposed by Brown and Riley [9] yields a faster propagating LEE than that of Joshi [10] in air. They also found that the transient boundary layer equations predict an even faster propagation rate for the LEE than that based on the criterion of Brown and Riley [9]. Herein we compare the rate of propagation of the LEE for vertical solid cylinders of several radii under various heat flux conditions using different criteria for two fluids, air ($Pr = 0.72$) and water ($Pr = 4.53$).

2. ANALYSIS

The steady state natural convection boundary layer equations adjacent to a vertical, heat generating cylinder (see inset of Fig. 3) for laminar, constant property, viscous flow with Boussinesq approximation are

$$\frac{\partial(RU)}{\partial X}+\frac{\partial(RV)}{\partial R}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial R}=T+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right) \tag{2}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial R}=\frac{1}{Pr}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) \tag{3}$$

subject to the following boundary and initial conditions

$$\begin{aligned} &U=0=V && \text{at } R=R_o && \text{for all } X\\ &U=0=T && \text{at } X=0 && \text{for all } R\\ &U\to 0,\ T\to 0 && \text{as } R\to\infty && \text{for all } X \end{aligned}$$

$$-\left.\frac{\partial T}{\partial R}\right|_{R=R_o}=1 \quad \text{for all } X \tag{4}$$

where

$$X=\frac{x}{r_o}R_o,\quad R=\frac{r}{r_o}R_o,\quad U=\frac{ur_o}{\nu R_o},\quad V=\frac{vr_o}{\nu R_o}$$

$$T=\frac{k(t-t_\infty)}{q''r_o}R_o,\quad Pr=\frac{\nu}{\alpha},\quad R_o=\frac{r_o}{(k\nu^2/g\beta q'')^{1/4}}$$

R_o being the fourth root of the modified Grashof number.

During the transient, one-dimensional conduction forms of equations (2) and (3) are

$$\frac{\partial U}{\partial \tau}=T+\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right) \tag{5}$$

$$\frac{\partial T}{\partial \tau} = \frac{1}{Pr}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) \quad (6)$$

subject to the following boundary and initial conditions

$$\begin{aligned} &U = 0 && \text{at } R = R_o && \text{for all } \tau \\ &U \to 0, T \to 0 && \text{as } R \to \infty && \text{for all } \tau \\ &U = 0 = T && \text{at } \tau = 0 && \text{for all } R \end{aligned}$$

$$Q\frac{\partial T_s}{\partial \tau} - \frac{\partial T}{\partial R}\bigg|_{R=R_o} = 1 \quad \text{for all } \tau \quad (7)$$

where

$$\tau = \frac{\hat{\tau}\nu}{r_o^2}R_o^2, \quad Q = \frac{c\nu}{kr_o}R_o, \quad T_s = \frac{k(t_s - t_\infty)R_o}{q''r_o}.$$

The boundary conditions in equation (7) imply that the temperature of the cylinder has been lumped in the radial direction. Justification for this assumption can be found in Velusamy and Garg [3]. Also, the effect of surface radiation can be neglected for a highly polished surface.

The penetration distance of LEE at any instant τ as proposed by Goldstein and Briggs [4] is

$$X_{GB}(\tau) = \max\left[\int_0^{\tau} U(\psi, R)\,d\psi\right] \quad (8)$$

where the velocity $U(\tau, R)$ is calculated from the solution of equations (5) and (6).

The penetration distance of LEE as proposed by Brown and Riley [9] is

$$X_{BR}(\tau) = \int_0^{\tau} \max\,[U(\psi, R)]\,d\psi. \quad (9)$$

The penetration distance of LEE obtained by applying the criterion of no overshoot in the mass flow rate during the one-dimensional process, proposed by Joshi [10], is X_J such that

$$\int_0^{R_\infty} U(\tau, R)R\,dR = \int_0^{R_\infty} U_{ss}(R, X_J)R\,dR. \quad (10)$$

The penetration distance of LEE obtained by applying the criterion of no overshoot in the maximum velocity during the one-dimensional process is X_U such that

$$\max\,[U(\tau, R)] = \max\,[U_{ss}(R, X_U)] \quad (11)$$

where the maximum is sought with respect to R.

The penetration distance of LEE obtained by applying the criterion of no overshoot in the shear stress during the one-dimensional process is X_S such that

$$\frac{\partial U}{\partial R}(\tau, R_o) = \frac{\partial U_{ss}}{\partial R}(R_o, X_S). \quad (12)$$

The penetration distance of LEE obtained by applying the criterion of no overshoot in surface temperature during the one-dimensional process is X_T such that

$$T(\tau, R_o) = T_{ss}(R_o, X_T). \quad (13)$$

In equations (10)–(13) subscript 'ss' represents steady state values obtained from the solution of equations (1)–(4).

3. SOLUTION

The boundary layer equations (1)–(3) subject to the boundary conditions (4) are solved by a finite difference marching technique. This technique is a modified form of the one described by Hornbeck [12] for flow through a circular pipe. While marching in the axial direction, the nonlinearity of the inertial terms and the interlinkage of momentum and energy equations are retained. Equations (5) and (6) subject to the boundary and initial conditions (7) are solved by a fully implicit finite difference technique. The finite difference form of equations (1)–(7) is solved iteratively by the Thomas Algorithm [12].

3.1. *Computational details*

Variable grid sizes were used in the axial and radial directions. Along the axial direction 184 grids were found sufficient to obtain a grid-independent solution. The grid size in the marching axial direction was 10^{-9} near the leading edge and was gradually increased to 2 near the downstream end. The number of grids in the radial direction was 171 for $R_o = 15$ and 551 for $R_o = 0.5$. Also, the smaller the value of R_o, the finer was the radial grid size near the surface of the cylinder in order to take care of increasing curvature effects. The radial grid size near the cylinder was 0.1 for $R_o = 15$ and 0.0125 for $R_o = 0.5$. The step size in time was gradually increased from 10^{-7} to 2 for $Pr = 0.72$ and from 10^{-7} to 0.2 for $Pr = 4.53$. A relaxation factor of 0.6 was used.

In order to remain in the laminar region, we restricted the calculation domain in the axial direction to $0 \leqslant X \leqslant 100$; $X = 100$ implies $Gr_x = g\beta q''x^4/(k\nu^2) = 10^8$. The thermal capacity, $\rho_m Cp_m$, of ordinary materials such as steel, nickel, copper, etc. is nearly the same. Hence $\rho_m Cp_m$ was not considered as a parameter in the present analysis. It can be shown that $Q = \rho_m Cp_m \nu R_o/2k$. Hence Q is directly proportional to R_o as well as to the cylinder radius r_o. If the curvature terms in equations (1)–(7) are dropped, the boundary layer and one-dimensional equations applicable to a flat plate are obtained. The results obtained by dropping these terms compare very well with the results of Joshi [10] for a flat plate. To check the numerical solution of one-dimensional equations (5)–(7), we compared the values of U and X_{GB} against the analytical solution of Goldstein and Briggs [4]. A maximum difference of 0.08% in U and 1.2% in X_{GB} was found. Further verification of the present numerical procedure is available in Velusamy and Garg [3].

4. RESULTS AND DISCUSSION

The rate of propagation of the LEE predicted by various criteria for $R_o = 0.5$ and $Pr = 0.72$ (air) are

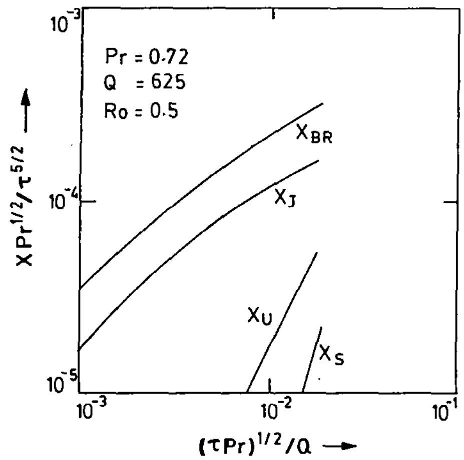

FIG. 1. Penetration distance of LEE on a vertical cylinder in air.

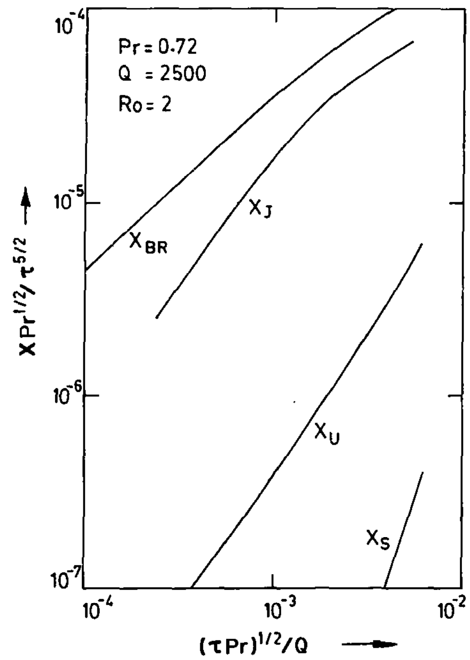

FIG. 2. Penetration distance of LEE on a vertical cylinder in air.

shown in Fig. 1. As already mentioned, the parameter R_o combines the dimensional radius of the cylinder (r_o) and heat flux (q''). Thus various values of R_o can be interpreted as (i) various values of r_o for a fixed q'', or (ii) various values of q'' for a fixed value of r_o. For example, when a steel (25% Cr, 20% Ni) cylinder is placed in air at about 70°C, $R_o = 0.5$ implies a radius (r_o) of 0.5 mm when the heat flux is 462 W m^{-2}. For this case the dimensional time required for LEE to reach $x = 100$ mm is 7.36 s as per equation (9). Figure 1 does not display the LEE propagation rate from equation (13) as it is several orders of magnitude less than the minimum value of the ordinate.

From Fig. 1, it is clear that the propagation rate due to equation (9) is the fastest of all. In other words, for cylinders, equation (9) predicts no overshoot in any of the physical quantities such as mass flow rate, surface shear stress, etc. during the one-dimensional process. However, for a flat plate, equations (8) and (9) predict an overshoot in the mass flow rate which is unrealistic for a Boussinesq fluid [10]. Amongst the criteria based on no overshoot in the physical quantities, the criterion of no overshoot in the mass flow rate yields the fastest and the criterion of no overshoot in the surface temperature yields the slowest rate of propagation of LEE. Also, the LEE propagation rate is faster when determined in order from equations (10)–(13). This order is the same as that observed by Joshi [10] for a flat plate. Similar propagation rates for $R_o = 2$ and 15 are shown in Figs. 2 and 3 for air. In these cases also, the criterion of Brown and Riley [9] yields the fastest LEE propagation rate followed by the criterion in equation (10). By a conversion of results to dimensional form, it was found that the speed of propagation of the LEE decreases with the heat flux, and as the cylinder becomes thicker.

The rate of propagation of LEE predicted by various criteria for $R_o = 0.5$ and $Pr = 4.53$ (water) are shown in Fig. 4. For example, $R_o = 0.5$ may imply a steel cylinder of radius 0.3125 mm when the heat flux is 462 W m^{-2}. It is clear from Fig. 4 that during the initial period $(\tau^{1/2}Pr^{1/2}/Q) < 1$, the criterion of equation (10) yields a faster propagation rate than others and is followed by the criteria in equations (9) and (11)–(13). This order is similar to that for a flat plate. But for larger time $(\tau^{1/2}Pr^{1/2}/Q) > 1$, different

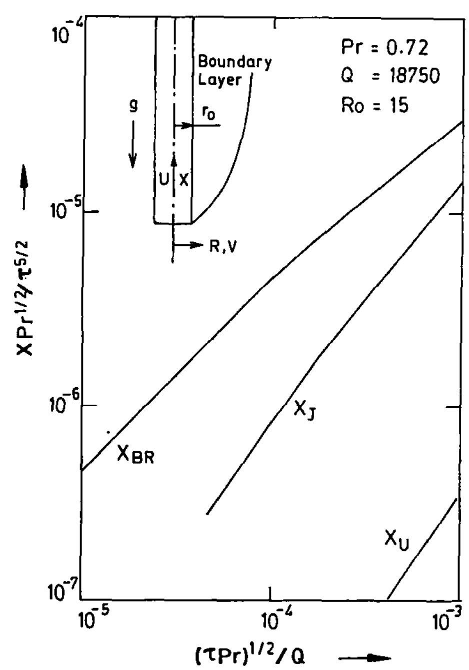

FIG. 3. Penetration distance of LEE on a vertical cylinder in air.

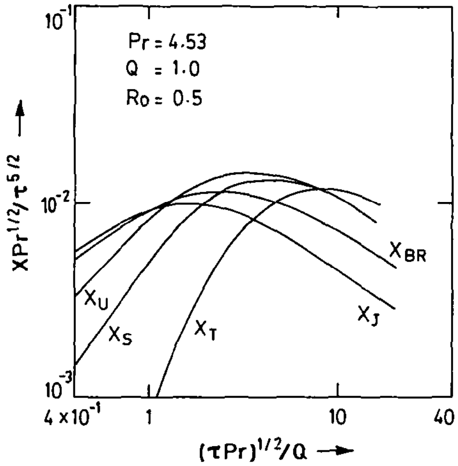

FIG. 4. Penetration distance of LEE on a vertical cylinder in water.

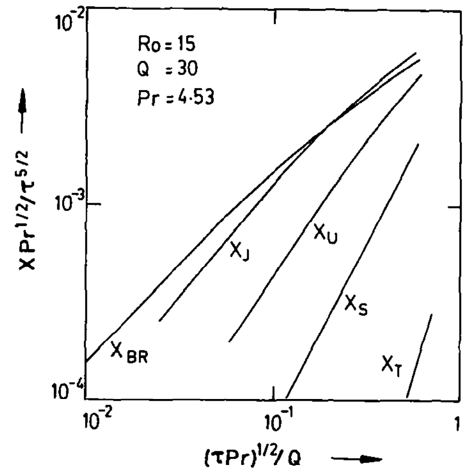

FIG. 6. Penetration distance of LEE on a vertical cylinder in water.

criteria yield faster propagation rates at different instants. For example, at $\tau^{1/2}Pr^{1/2}/Q = 2$, the criterion of equation (11) is the fastest and that of equation (13) is the slowest, but at $\tau^{1/2}Pr^{1/2}/Q = 15$, criterion of equation (10) is the slowest and that of equation (13) is the fastest. Thus, there is no unique criterion that always yields the fastest rate of propagation of LEE. This behavior is not observed for a flat plate as well as for a cylinder in air. Also, all curves display a maximum while this is not true for a flat plate.

The rate of propagation of LEE for $R_o = 2$ and 15 when $Pr = 4.53$ are presented in Figs. 5 and 6. In these cases also, there is no unique criterion that predicts the fastest propagation rate for LEE, and the results are similar to those in Fig. 4 for $R_o = 0.5$. Contrary to the observation made for air, there is no monotonic change in the propagation rate of LEE with the radius of the cylinder. Instead, it passes through a maximum as the radius of the cylinder increases, reaching the maximum at $R_o = 2.8$ (for $\hat{\tau}_{BR}$), as seen following

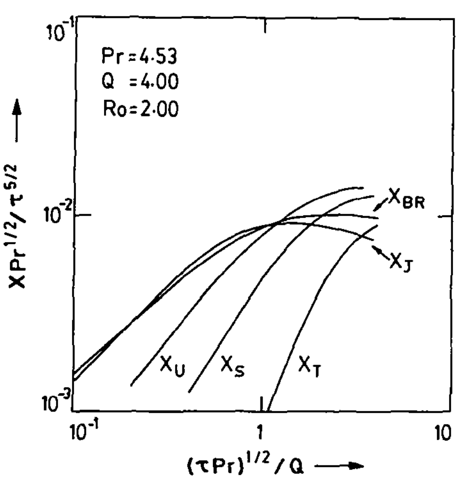

FIG. 5. Penetration distance of LEE on a vertical cylinder in water.

the conversion of results to dimensional form. This behavior is also exhibited by other criteria. However, the value of R_o where the rate attains a maximum value differs from one criterion to the other. It was found that for identical conditions, the rate of propagation of LEE is slower in water than in air, as expected due to the larger flow vigor in flows in air.

5. CONCLUSIONS

The rate of propagation of the leading edge effect predicted by five different criteria are compared for vertical solid cylinders of various radii and heat flux conditions in air and water. The following conclusions are drawn for the parameter values studied:

(a) For cylinders in water there is no unique criterion that would always result in the fastest rate of propagation of LEE. This is contrary to the observation for a flat plate.

(b) In the case of air, the thicker the cylinder, the slower is the rate of propagation of LEE based on any criterion.

(c) In the case of water, the rate of propagation of LEE displays a maximum when plotted against the cylinder radius.

(d) The rate of propagation of LEE is slower in water than in air, as expected.

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